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SPONTANEOUS SWIRLING IN MHD FLOWS WITH CIRCULAR STREAMLINES

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This paper considers the problem of the evolution of azimuthal perturbations in axisymmetric magnetohydrodynamic flows of an ideally conducting inviscid fluid with circular streamlines. The fluid is in a toroidal gap between two surfaces with constant values of the stream function. The equations of fluid motion are derived in the approximation of infinitely a narrow gap. The parameters at which spontaneous swirling is possible are determined numerically, and the properties of secondary swirling flows resulting from instability of the initial steady-state poloidal flow are established.

Key words: *axisymmetric magnetohydrodynamic vortex with closed streamlines, stability, numerical calculation of the evolution of azimuthal perturbations, spontaneous swirling.*

Introduction. Let us define spontaneous swirling. We consider steady-state axisymmetric (poloidal) flow of an inviscid fluid in a bounded axisymmetric region sustained by the field of mass forces (axisymmetric, poloidal). In cylindrical coordinates, we have

$$\boldsymbol{r} = (z, r, \varphi), \qquad \boldsymbol{v}_0(z, r) = (w_0(z, r), u_0(z, r), 0).$$

Small axisymmetric perturbations are introduced to this flow (generally, arbitrary) with nonzero azimuthal ($v_\varphi = v$) velocity component. The mass-force field and the boundaries are not perturbed. If the perturbations damp or their amplitude does not increase, the flow is stable and swirling does not arise. In the case of instability, the perturbations increase. If the evolution of the initial perturbations, by virtue of the exact nonlinear equations, gives rise to flow (steady-state, periodic, unsteady chaotic, turbulent) in which the average azimuthal velocity component is finite:

$$\langle v_\varphi \rangle = \int_0^{2\pi} v_\varphi(t, r, z, \varphi)\, d\varphi \neq 0,$$

and the energy of the rotational motion around the symmetry axis is comparable to the energy of the initial poloidal flow, we will speak of the occurrence of spontaneous swirling.

The problem of spontaneous swirling was first formulated in [1] as follows: can axisymmetric rotational flow arise in the absence of obvious external sources of rotation, i.e., under conditions where axisymmetric motion without rotation is obviously possible?

A simple example of swirling is a bath tube vortex [2]. In this case, the mechanism generating rotational motion, just as that of intense mesoscale atmospheric vortexes (dust poles, whirlwinds, tornados), is not completely understood. It is not impossible that spontaneous swirling plays an important role in this mechanism. The problem of spontaneous swirling was studied in [1–10], where results of investigation of this phenomenon are given, the formulation of the problem is refined, and its various versions are given. Another treatment of the spontaneous swirling problem is considered in [3]. The difference between this treatment and that given above is discussed in [10].

At present, most of the studies of this topic have been performed in a linear formulation. The questions of the effect of nonlinearity and the nature (structure, intensity) of the secondary flow resulting from instability remain open.

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The present paper considers the problem of the stability and evolution of azimuthal perturbations in the supercritical region in the case of axisymmetric magnetohydrodynamic (MHD) flows of an ideally conducting inviscid fluid in a nonlinear formulation. The toroidal region is bounded by circular streamlines. The approximation of an infinitely narrow gap is used. The equations obtained by passage to the limit are solved numerically. The parameter values at which spontaneous swirling is possible and the characteristics of the secondary swirling flows resulting from instability of the initial poloidal flow are determined.

1. Formulation of the Problem. In the conventional notation, flows of an ideally conducting inviscid incompressible fluid in a magnetic field are described by the following system of equations (fluid density $\rho = 1$):

$$\boldsymbol{v}_t - \boldsymbol{v} \times \operatorname{rot} \boldsymbol{v} + \boldsymbol{h} \times \operatorname{rot} \boldsymbol{h} = \boldsymbol{F} - \nabla(p + \boldsymbol{v}^2/2); \tag{1.1}$$

$$\operatorname{div} \boldsymbol{v} = 0; \tag{1.2}$$

$$\boldsymbol{h}_t = \operatorname{rot} (\boldsymbol{v} \times \boldsymbol{h}); \tag{1.3}$$

$$\operatorname{div} \boldsymbol{h} = 0. \tag{1.4}$$

Here $\boldsymbol{h} = \boldsymbol{H}/\sqrt{4\pi}$; $\boldsymbol{F} = (F_z, F_r, 0)$ is the poloidal field of the external mass forces.

Using (1.2) and (1.4), the poloidal components of the velocity and magnetic field $\boldsymbol{v} = (u, v, w)$, $\boldsymbol{h} = (h_1, h, h_3)$ of the examined steady-state axisymmetric rotational fluid flows in cylindrical coordinates $\boldsymbol{r} = (r, \varphi, z)$ can generally be described by the relations

$$u = -\frac{\gamma(\psi)}{r}\frac{\partial \psi}{\partial z}, \qquad w = \frac{\gamma(\psi)}{r}\frac{\partial \psi}{\partial r}, \qquad h_1 = -\frac{\varepsilon(\psi)}{r}\frac{\partial \psi}{\partial z}, \qquad h_3 = \frac{\varepsilon(\psi)}{r}\frac{\partial \psi}{\partial r},$$

where $\gamma(\psi)$ and $\varepsilon(\psi)$ are arbitrary dependences on the stream function ψ. Below, we consider flows in which the functions $\gamma(\psi)$ and $\varepsilon(\psi)$ are constant, and, hence, without loss of generality, one of them can be set equal to unity. Let $\gamma = 1$. Then, ε acquires the meaning of the proportionality coefficient between the poloidal components of the magnetic field and the velocity $\boldsymbol{h}_p = \varepsilon \boldsymbol{v}_p$ in the initial poloidal flow. The quantity ε will be called magnetization.

In the problem of spontaneous swirling, the source of the main (initial) poloidal flow is not so important; therefore, for the description of the initial axisymmetric flow, the stream function can be taken to be any fairly regular function $\psi(r, z)$ that assumes constant values on closed lines in the meridional plane (r, z). Any such line can be chosen as the boundary of the toroidal field. In the presence of the corresponding mass forces and motion of the boundaries, any flow defined in such a way can be treated as an exact steady-state axisymmetric solution of the MHD equations.

Investigation of the stability of such flows using both analytical and numerical methods is a difficult problem. Therefore, to determine the main regularities in the problem of swirling in flows with closed streamlines, along with investigation of the problem for a Hill–Shafranov vortex in the exact (linear) formulation for inviscid and viscous fluids, we performed stability analysis for flows which are exact steady-state solutions in the sense specified above in the approximate formulation (a narrow gap in flow with circular streamlines) [9, 10].

2. MHD Flows with Circular Streamlines. Let the stream function have the form $\psi(z, r) = \psi(R)$, where $R = \sqrt{(r - r_0)^2 + z^2}$ is the distance from the circular axis of a torus ($R < r_0$); r_0 is the distance from the z axis to the general circular axis of toruses of small radius R. This flow can be treated as an exact steady-state solution of the MHD equations in the sense specified above.

Let us introduce the azimuthal and radial poloidal components of the velocity vector. In the initial flow, the velocity components are given by

$$q(R, \theta) = w \cos\theta + u \sin\theta = -\frac{1}{r_0 - R\cos\theta}\frac{d\psi(R)}{dR}, \qquad p(R, \theta) = w \sin\theta - u \cos\theta = 0,$$

and the magnetic-field components are

$$g(r, \theta) = h_3 \cos\theta + h_1 \sin\theta = -\frac{\varepsilon}{r_0 - R\cos\theta}\frac{d\psi(R)}{dR},$$

$$f(R, \theta) = h_3 \sin\theta - h_1 \cos\theta = 0.$$

In the new variables (R, θ), system (1.1)–(1.4) is written as

$$q_t + pq_R + \frac{q}{R}\, q_\theta + \frac{pq}{R} - s(R,\theta)v^2 = -\frac{1}{R}\,\Pi_\theta + fg_R + \frac{g}{R}\, g_\theta + \frac{fg}{R} - s(R,\theta)h^2 + F_z \cos\theta + F_r \sin\theta; \tag{2.1}$$

$$p_t + pp_R + \frac{q}{R}\, p_\theta - \frac{q^2}{R} + c(R,\theta)v^2 = -\Pi_R + ff_R + \frac{g}{R}\, f_\theta - \frac{g^2}{R} + c(R,\theta)h^2 - F_r \cos\theta + F_z \sin\theta; \tag{2.2}$$

$$v_t + pv_R + \frac{q}{R}\, v_\theta + (s(R,\theta)q - c(R,\theta)p)v = fh_R + \frac{g}{R}\, h_\theta + (s(R,\theta)g - c(R,\theta)f)h; \tag{2.3}$$

$$g_t + pg_R + \frac{q}{R}\, g_\theta + \frac{pg}{R} = fq_R + \frac{g}{R}\, q_\theta + \frac{fq}{R}; \tag{2.4}$$

$$f_t + pf_R + \frac{q}{R}\, f_\theta = fp_R + \frac{g}{R}\, p_\theta; \tag{2.5}$$

$$h_t + ph_R + \frac{q}{R}\, h_\theta - (s(R,\theta)q - c(R,\theta)p)h = fv_R + \frac{g}{R}\, v_\theta - (s(R,\theta)g - c(R,\theta)f)v; \tag{2.6}$$

$$(Rp)_R + q_\theta + (s(R,\theta)q - c(R,\theta)p)R = 0; \tag{2.7}$$

$$(Rf)_R + g_\theta + (s(R,\theta)g - c(R,\theta)f)R = 0. \tag{2.8}$$

Here

$$s(R,\theta) = \frac{\sin\theta}{r_0 - R\cos\theta}, \qquad c(R,\theta) = \frac{\cos\theta}{r_0 - R\cos\theta}.$$

The flow considered is enclosed between two streamlines corresponding to the values $R = R_0$ and $R = R_0 + d$. The solution of system (2.1)–(2.8) is subject to the following boundary conditions: all required functions $\tilde{F}(R,\theta)$ are periodic on the streamlines: $\tilde{F}(R,0) = \tilde{F}(R,2\pi)$; the radial components p and f vanish on the boundaries of the region considered: $p(R_0,\theta) = f(R_0,\theta) = 0$ and $p(R_0 + d,\theta) = f(R_0 + d,\theta) = 0$.

With the choice of an appropriate poloidal field of the mass forces, system (2.1)–(2.8) has exact steady-state solutions (in the sense specified above), in which $\psi(z,r) = \psi(R)$ is an arbitrary function of the variable R. The question arises: what will occur if small axisymmetric perturbations are introduced into the flow? This question can be answered by solving the Cauchy problem for system (2.1)–(2.8). This problem can be solved numerically, but it is very difficult and requires powerful computing resources. In the present paper, we propose a simplified mathematical model obtained from the exact model by passing to the limit of an infinitely narrow gap between the boundary streamlines.

3. Approximation of an Infinitely Narrow Gap between the Boundary Streamlines. Let the flow be enclosed in a region $R_0 < R < R_0 + d$. We consider passage to the limit such that $d \to 0$, $\rho \to \infty$, and $\rho d \to \rho_*$, where ρ_* is a finite quantity (mass per unit area). We next assume that $\rho_* = 1$. In this case, $|\boldsymbol{H}| \to \infty$, $\Pi \to \infty$, and $|\boldsymbol{h}| = |\boldsymbol{H}|/\sqrt{4}\,\pi\rho \to |\boldsymbol{h}|_*$ and $\Pi d \to \Pi_*$ are finite quantities (Π_* is the negative surface tension). The difference $\Pi(R_0 + d) - \Pi(R_0) \to \Delta\Pi$ is also a finite quantity (the pressure difference providing centrifugal acceleration). The velocity and magnetic-field components $p \to 0$ and $f \to 0$.

From continuity equations (2.7) and (2.8), it follows that in the limit,

$$q(t,\theta) = \frac{Q(t)}{1 - k\cos\theta}, \qquad Q(0) = Q_0,$$
$$g(t,\theta) = \frac{G(t)}{1 - k\cos\theta}, \qquad G(0) = \varepsilon Q_0, \tag{3.1}$$

where $k = R_0/r_0$.

From Eq. (2.4), it follows that in the limit

$$g_t + \frac{q}{R_0}\, g_\theta = \frac{g}{R_0}\, q_\theta$$

and, taking into account (3.1),

$$g(t,\theta) = \frac{\varepsilon Q_0}{1 - k\cos\theta}.$$

By passing to the limit, we have

$$-\frac{q^2}{R_0} + c(R_0,\theta)v^2 = -\Delta\Pi - \frac{g^2}{R_0} + c(R_0,\theta)h^2; \tag{3.2}$$

$$q_t + \frac{q}{R_0}\, q_\theta - s(R_0,\theta)v^2 = -\frac{1}{R_0}\, \Pi_{*\theta} + \frac{g}{R_0}\, g_\theta - s(R_0,\theta)h^2; \tag{3.3}$$

$$v_t + \frac{q}{R_0}\, v_\theta + s(R_0,\theta)qv = \frac{g}{R_0}\, h_\theta + s(R_0,\theta)gh, \qquad h_t + \frac{q}{R_0}\, h_\theta - s(R_0,\theta)qh = \frac{g}{R_0}\, v_\theta - s(R_0,\theta)gv.$$

Equation (3.2) defines the pressure difference $\Delta\Pi$ which provides the centrifugal acceleration.

Integration of (3.3) over the period yields an integral equation which defines the time dependence of the quantity $Q(t)$ and differential equations for the azimuthal components of the velocity field and magnetic field in the dimensionless variables (time is normalized by the quantity R_0/q_0, and the velocity and magnetic-field components v and h are normalized by the quantity q_0):

$$\begin{gathered} Q'(t) = \frac{k\sqrt{1-k^2}}{2\pi} \int\limits_0^{2\pi} \frac{\sin\theta}{1 - k\cos\theta}\,(v^2 - h^2)\,d\theta, \qquad Q(0) = 1(Q_0), \\ v_t + \frac{Q(t)}{1 - k\cos\theta}\, v_\theta + \frac{Q(t)k\sin\theta}{(1 - k\cos\theta)^2}\, v = \frac{\varepsilon}{1 - k\cos\theta}\, h_\theta + \frac{\varepsilon k\sin\theta}{(1 - k\cos\theta)^2}\, h; \end{gathered} \tag{3.4}$$

$$h_t + \frac{Q(t)}{1 - k\cos\theta}\, h_\theta - \frac{Q(t)k\sin\theta}{(1 - k\cos\theta)^2}\, h = \frac{\varepsilon}{1 - k\cos\theta}\, v_\theta - \frac{\varepsilon k\sin\theta}{(1 - k\cos\theta)^2}\, v. \tag{3.5}$$

The integrodifferential system (3.4) obtained by the passage to the limit is much simpler than the initial systems (2.1)–(2.8).

4. Conservation Laws. For the integrodifferential system (3.4), (3.5), the following conservation laws are valid: the energy conservation law

$$\int\limits_0^{2\pi} \frac{1 - k\cos\theta}{2}\,(v^2 + h^2 + q^2)\,d\theta = \text{const},$$

the angular momentum conservation law

$$\int\limits_0^{2\pi} (1 - k\cos\theta)^2 v\,d\theta = \text{const},$$

the magnetic flux conservation law

$$\int\limits_0^{2\pi} h\,d\theta = \text{const}.$$

In addition, the following relation holds:

$$\int\limits_0^{2\pi} (1 - k\cos\theta)^2 vh\,d\theta + \frac{2\pi\varepsilon}{\sqrt{1-k^2}}\, Q(t) = \text{const}.$$

In the numerical calculations, these relations were used to check the calculation accuracy.

5. Steady-State Flows. The system considered has a steady-state solution

$$q(t,\theta) = \frac{Q_0}{1 - k\cos\theta}, \qquad g(t,\theta) = \frac{\varepsilon Q_0}{1 - k\cos\theta},$$

$$v = \frac{a}{1 - k\cos\theta} + \varepsilon b(1 - k\cos\theta), \qquad h = \frac{\varepsilon a}{1 - k\cos\theta} + b(1 - k\cos\theta),$$

where a and b are arbitrary constants. With an appropriate choice of values of the arbitrary constants, this solution describes analogs of the well-known steady-state solutions in the exact formulation: Hill vortex, Higgs vortex (Hill swirling vortex), Hill–Shafranov MHD vortex, and Higgs MHD vortex (MHD swirling vortex). For the last case, an exact analytical solution is obtained in [11].

Within the framework of the model of an ideally conducting inviscid fluid, the obtained solution is isolated from the set of unsteady solutions.

6. Linear Stability. The linear stability of steady-state axisymmetric MHD flows with closed streamlines was studied in [9]. From the results of that work, it follows that in the case considered, if the condition

$$1 > \varepsilon > \frac{1}{\sqrt{c}}, \qquad c = \frac{1}{4\pi^2}\int_0^{2\pi}(1 - k\cos\theta)^3\,d\theta\int_0^{2\pi}(1 - k\cos\theta)^{-1}\,d\theta = \frac{1 + (3/2)k^2}{\sqrt{1 - k^2}}$$

is satisfied, the initial flow is monotonically unstable. This condition is sufficient but not necessary. Nonmonotonic instability can also take place for smaller values of k (for given ε) for which the indicated inequality is valid. Determination of the exact boundary in the plane (ε, k) requires a large volume of calculations. However, to draw a justified conclusion on the possibility of spontaneous swirling, it is necessary to consider the initial perturbations satisfying the initial condition $h = 0$ [8]. If the completeness of the eigenfunctions is not proved, the spectral method does not permit to draw such a conclusion. With the additional condition specified above, the region of instability can change significantly (or even disappear); therefore, numerical calculations of the problem with initial conditions in the supercritical fields are required to confirm the existence of instability above the critical value of k.

7. Numerical Calculation. To study the evolution of the initial perturbations, it is convenient to rearrange the obtained system as follows. Setting

$$A = v + h, \qquad B = v - h.$$

Then, for A and B, we obtain

$$Q'(t) = \frac{k\sqrt{1 - k^2}}{2\pi}\int_0^{2\pi}\frac{\sin\theta}{1 - k\cos\theta}\,AB\,d\theta, \qquad Q(0) = 1; \tag{7.1}$$

$$A_t + \frac{Q(t) - \varepsilon}{1 - k\cos\theta}\,A_\theta + \frac{(Q(t) + \varepsilon)k\sin\theta}{(1 - k\cos\theta)^2}\,B = 0,$$

$$B_t + \frac{Q(t) + \varepsilon}{1 - k\cos\theta}\,B_\theta + \frac{(Q(t) - \varepsilon)k\sin\theta}{(1 - k\cos\theta)^2}\,A = 0.$$

The initial velocity perturbation introduced into the initial steady-state flow is specified in the form

$$A(0,\theta) = B(0,\theta) = A_0 + A_1\sin f\theta,$$

which corresponds to the perturbation

$$v(0,\theta) = A_0 + A_1\sin f\theta, \qquad h(0,\theta) = 0$$

(A_0 and A_1 are some constants which are small compared to unity; f is an integer).

Solutions of system (7.1) were found numerically from the initial data and the periodicity conditions $A(0) = A(2\pi)$ and $B(0) = B(2\pi)$. The calculations used an explicit four-point difference scheme of second-order approximation $O(\tau^2 + \delta^2)$ [12].

The numerical calculations in the supercritical region showed that perturbations lead to spontaneous swirling (in the above sense), i.e., flow in which the intensity (energy) of rotational motion is comparable to the intensity (energy) of the initial poloidal flow.

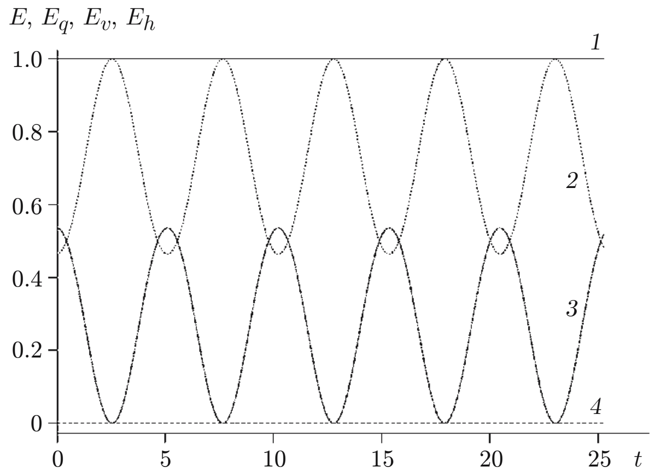

Fig. 1. Time dependences of the energy for $\varepsilon = 0$, $k = 0.5$, and $A_0 = 1$: 1) total energy E; 2) azimuthal energy E_v; 3) poloidal energy E_q; 4) magnetic-field energy E_h.

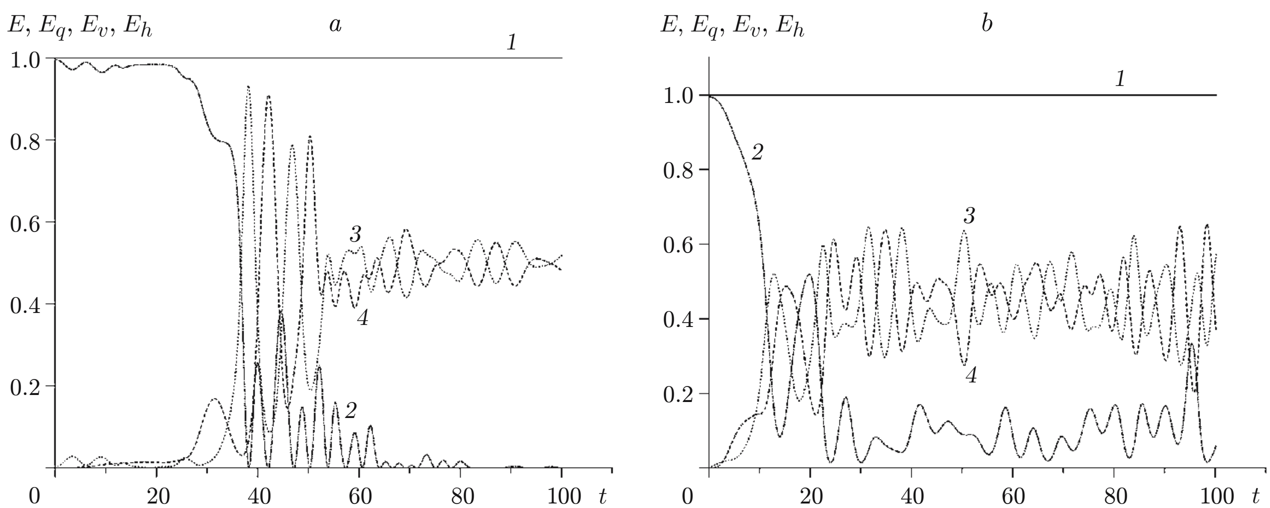

Fig. 2. Time dependences of the energy for $k = 0.5$, $A_1 = 0.1$, and $f = 1$: (a) $\varepsilon = 0.05$; (b) $\varepsilon = 0.5$; the remaining notation is the same as in Fig. 1.

Figures 1–3 gives calculation results for various values of the parameters ε and k. Figure 1 shows time dependences of the azimuthal E_v, poloidal E_q, and total E energies (normalized by the total initial energy) for $\varepsilon = 0$ and $k = 0.5$. In this case, magnetic field is absent (magnetic-field energy $E_h = 0$) and the total energy conservation law is satisfied fairly exactly ($E = 1$). For $\varepsilon = 0$, the initial poloidal flow is stable and the flow arising under any initial conditions is periodic. As the initial energy of the azimuthal perturbation increases, the period also increases.

Figure 2 gives the same dependences for $\varepsilon = 0.05$; 0.50, $k = 0.5$, and $v(0, \theta) = 0.1 \sin\theta$. Figure 3 shows curves of $v(\theta)$ and $h(\theta)$ for $t = 100$. In these cases, the spatial period 2π over θ was divided into $N = 800$ meshes. From Figs. 2 and 3, it follows that the energy conservation law is satisfied with sufficient accuracy ($E = 1$). The conservation laws for the angular momentum and the azimuthal magnetic flux are also satisfied fairly exactly. For the chosen initial conditions, these flows are equal to zero, and during calculation, they differ from zero by not more than 0.001.

The results show that due to instability for $\varepsilon \neq 0$, the energy of the azimuthal components of the velocity and magnetic field increases and the poloidal-flow energy decreases to almost zero (at $\varepsilon = 0.05$). The time-averaged

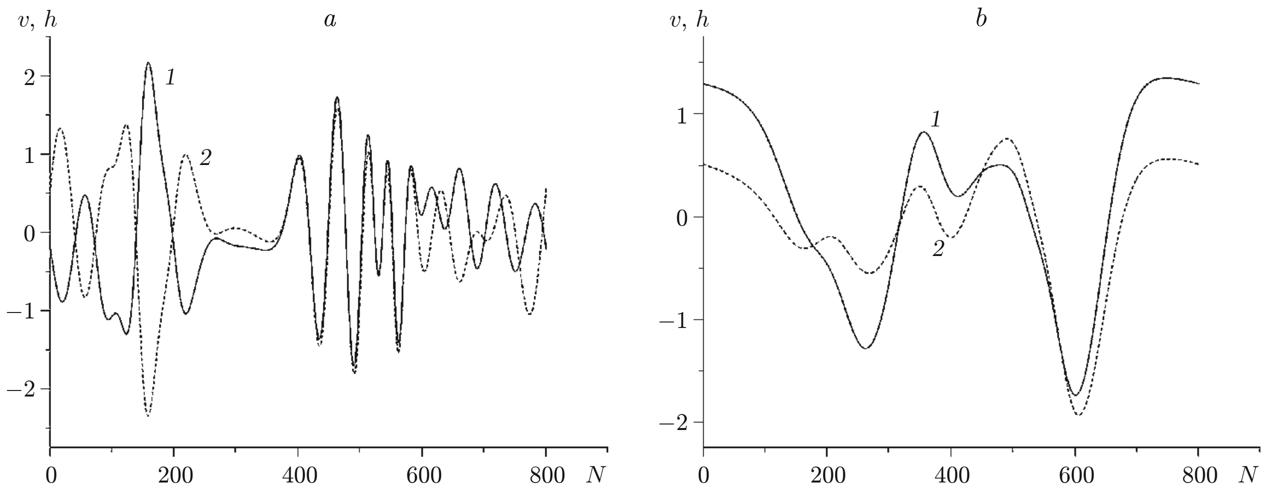

Fig. 3. Dependences $v(N)$ (1) and $h(N)$ (2) for $k = 0.5$, $A_1 = 0.1$, and $f = 1$: (a) $\varepsilon = 0.05$; (b) $\varepsilon = 0.5$.

energies of the magnetic field and rotational fluid motions for large values of t become comparable in magnitude to the initial energy of the initial poloidal flow. Thus, spontaneous swirling (according to the above definition) arises.

The flow resulting from instability is irregular (chaotic) in both time and space. The amplitudes of short-wave harmonics increase in time.

Conclusions. The results of the study suggest that spontaneous swirling is possible in a bounded region in a rigid (guaranteed absence of the axial angular momentum influx) by fairly natural formulation.

It is shown that for certain parameters of the initial poloidal flow (degree of magnetization ε and geometrical characteristic k), the energy of the initial flow is almost completely converted to the energy of the azimuthal (rotational) velocity field and magnetic field that arise. In this case, over a wide range of parameters of the initial flow, the time-averaged energies of the rotational motion and magnetic field take identical values for large t.

We note that even for a small degree of magnetization $\varepsilon = 0.05$, instability can give rise to a considerable magnetic field, which can be treated as the spontaneous occurrence of a magnetic field due to extension of the force lines of the initial weak poloidal magnetic field. It is not impossible that the same mechanism plays an important role in the geomagnetic dynamo phenomenon.

The numerical studies revealed some other properties of the secondary flow: for $\varepsilon = 0$, the initial flow is stable and the flow arising for any initial conditions (not small) is periodic; in the case of a high degree of supercriticity for $0 < \varepsilon < 1$, the flow is irregular (chaotic) in time and its spatial structure is rather complex: the presence of differential rotation, which is a consequence of the conservation laws for the angular momentum and azimuthal magnetic flux; the number of maxima in the spatial distribution of the amplitudes v and h increase (apparently, without bound) with increasing time.

Thus, the present work shows the possibility of spontaneous swirling with nonlinearity taken into account. In the formulation (model) considered, the fluid viscosity is ignored. Investigation [10] of the linear stability of Hill–Shafranov MHD vortex and MHD flow in a narrow (but finite) gap between two circular streamlines for a viscous fluid showed the possibility of swirling. However, the question of the possibility of spontaneous swirling in a bounded region taking into account nonlinearity and viscosity remains open.

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